Robustness of Positive Real Controllers for Large Space Structures

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The robustness of a continuous positive real controller design is established by linking the positivity theory to the standard singular value robustness tests. By application of the singular value test to a model of the deviation from positivity induced by actuators, computational delays, etc., the global stability of the control design can be assured, even with significant modeling errors due to modal uncertainty. Both theoretical and experimental verification of this stability result is presented.

I. Introduction

THE design of controls for large space structures (LSS) is a difficult task due to the complexity of these dynamical systems. The structural model of these flexible bodies has a large number of vibration modes that are widely spaced in frequency and are characterized by a very low inherent damping. Generally, the dynamic model is poorly defined, leading to considerable uncertainty (especially in the higher modes) in the modal frequencies, damping, and mode shapes. In addition, the control design model is usually based on a low-order modal representation of the finite element model to reduce the complexity and sensitivity of the control design. This modal truncation leads to observation and control spillover that can destabilize one or more of the poorly damped modes. Controllers are expected to be implemented using digital systems, although many analysis techniques use only a continuous system theoretical approach. All of these approximations used in the process of control design make the robustness of the control laws a matter of particular interest for the control system designer.

The existing literature on the space structure control problem is extensive (see, e.g., Refs. 1-4). Numerous investigators have proposed techniques for dealing with the uncertainties inherent in the structural dynamic model. A powerful technique that can mathematically guarantee stability in the presence of significant modeling uncertainty is the positive real approach,[5,6] which guarantees stability of feedback systems that satisfy the positive real criteria. (See the sequel for a discussion of these criteria.) The positivity approach has been used to (theoretically) design stable controls for spacecraft models that give good transient performance, are robust to parameter variations in the spacecraft structural model, and are insensitive to failures in multiple actuators and/or sensors.[6]

A fundamental limitation remains, however, in the strictness of the positive real property, a mathematical condition that can be satisfied for ideal sensors and actuators but that fails for real components due to phase lag, excessive high frequency attenuation, and computational delays. In fact, it is easily shown that no digitally controlled continuous system can satisfy the fundamental positive real stability theorem.[7]

Currently in the controls field, the robustness of multivariable systems is one of the major research areas. The seminal article by Doyle and Stein[8] showed the importance of the singular values of the return difference matrix $\sigma(I+GH)$ for investigating robustness of various perturbative models. A succession of articles by Doyle and other researchers[9-12] has led to an abundance of information on structured and unstructured perturbations, stability analyses, and even design procedures to ensure robustness.

Unfortunately, the form of the perturbation models for the conventional unstructured perturbation are inappropriate for the large space structure model and generally fail to give significant results. For example, in a design for the Draper II space telescope, Kissel and Hegg[13] were unable to achieve desired stable performance due to the modal perturbations encountered. The essence of the problem lies in the nature of the assumed perturbations. The conventional singular value analysis is oriented toward high frequency unstructured perturbations in magnitude and phase. LSS control has these problems caused by the sensors and actuators, but the modal uncertainty is not of this form. A possible approach is to use a structured perturbation model and, in particular, to use real perturbations to capture this problem. Current work in this direction has been reported by Doyle,[12] Jones,[14] Tahk and Speyer,[15] and Yedavelli,[16] yet much remains to be done before a general

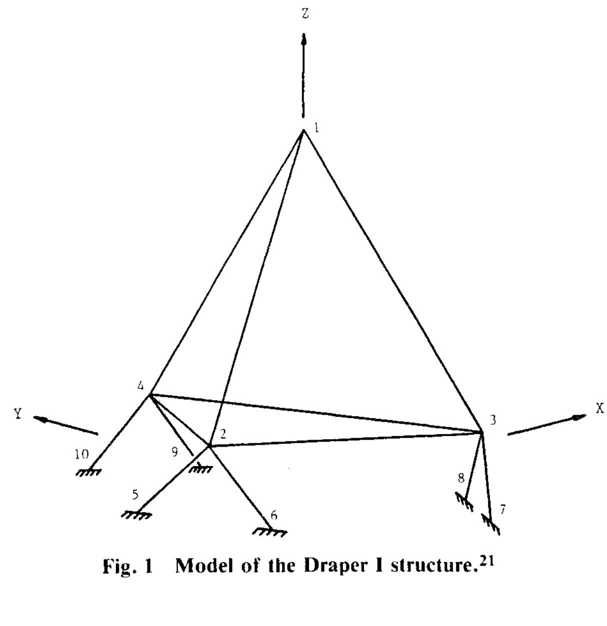

Fig. 1 Model of the Draper I structure.[21]

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theory is complete. Much of the difficulty encountered in the robustness research is caused by the generality of the systems that must be considered. By focusing on the structural control problem, we can take advantage of the special mathematical structure of this problem to yield important results.

For large space structures the dynamic model is characterized by a large number of "significant" structural modes in a frequency range that is within the desired bandwidth of the system. These modes are characterized by fairly low damping, and the primary goal of the controller is to add damping to these modes. Mathematical models of these systems are generally constructed from a finite element representation or from a simplified continuum approach. In either case it has been common knowledge that while the first few low frequency modes can be predicted fairly well, higher frequency modes are extremely difficult to predict accurately. In addition, these modes may be extremely sensitive to small parameter changes (e.g., stiffness, mass) that may be expected to occur over the life of the structure. Finally, in terms of constructing a control design model, it is often not the lowest frequency modes that are of interest. Often these low frequency modes may be associated with the vibration of appendages or other "local modes." Alternately, modes may be relatively uncontrollable by the system inputs or unobservable in the system outputs. Such modes, while not contributing greatly to the total system response, can have a large effect on the maximum singular value of the perturbation matrix $\bar{\sigma}(L)$ at the unmodeled model frequency, warning of a potential stability problem. Mathematically this is correct—a small (unstructured) change in the system dynamics at this frequency could cause the system poles to go unstable. Nevertheless, since this mode is within the control system bandwidth, the conventional approach of reducing system gains at this frequency is not appropriate.

For higher frequencies the unmodeled mode problem remains serious, although outside the control bandwidth, stability robustness can be assured through high frequency gain attenuation. This permits the unstructured singular value bounds to be met as long as unmodeled modal damping is not arbitrarily small. Another source of modeling errors (particularly) at high frequency is in the area of actuator dynamics. In most preliminary analyses, actuation dynamics are neglected. Tests of "proof mass" type actuators indicate that significant amounts of phase shift are often present as well as nonlinear effects due to friction and hysteresis. As long as this uncertainty is restricted to a higher frequency range, then the unstructured singular value analysis can accommodate this error source.

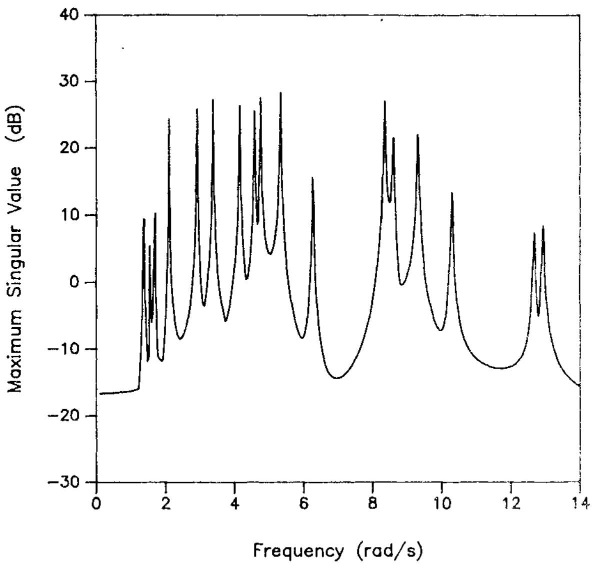

Fig. 2 Lm(ω) for a typical perturbation of the Draper I spacecraft model.

For low frequency error sources, the primary conclusion to be drawn is that there must be additional information available about the system if some stability robustness is to be ensured. Our work in the area of positive real controls indicates that this approach offers additional robustness criteria not available through unstructured singular value analysis. In this paper we outline the theory of positive real control (Sec. II) and the standard robustness results (Sec. III). Finally, in Sec. IV we propose a methodology to combine these analyses to produce meaningful stability results, and in Sec. V we demonstrate theoretical and experimental results, verifying the robustness of the positive real laws.

II. Review of Positivity

The formal definition of a positive real system is as follows.[17]

Definition 1: Positive Real System

1) A system with $m \times m$ transfer matrix $G(s)$ is positive real if

(i) $G(s)$ is real for real s and is analytic in the right half plane.

(ii) For $Re(s) > 0$, then

$$\left[G(s) + G^*(-s)\right] \geq 0 \tag{1}$$

where ( )* means conjugate transpose and the inequality in Eq. (1) means that the matrix on the left side of Eq. (1) is non-negative definite.

2) Equivalent to 1) are the following conditions:

(i) $G(s)$ is real for real s and is analytic in the right half plane.

(ii) Poles of $G(s)$ on the imaginary axis are simple and such that the residue matrix is non-negative definite Hermitian.

(iii) For all real ω with $s = j\omega$ not a pole of an element of $G(s)$, then

$$\left[G(j\omega) + G^*(j\omega)\right] \geq 0$$

If these inequalities are strictly satisfied (>), then the system is referred to as "strictly positive real." In network theory, positive realness is commonly related to a property of passive networks; viz., that the system is dissipative in that the (generalized) energy in the system at time t, minus the energy added to the system by the input function, is bounded (above) by the initial energy. (See Anderson.[18]) Thus positive realness implies a rather strong type of stability.

The test for positive realness in Definition 1 is seen to be fairly restrictive in terms of the type of transfer functions commonly encountered in engineering. For a scalar transfer function to be positive real, the Nyquist plot of $G(j\omega)$ must remain in the first and fourth quadrants, i.e., it must exhibit less than ± 90 deg of phase shift at all frequencies. Thus, the transfer functions $1/(s+p)$ and $s/(s^2 + \zeta\omega_n s + \omega_n^2)$ can easily be shown to be positive real. The transfer function $\omega_n^2/(s^2 + 2\zeta\omega_n s + \omega_n^2)$ is seen to be not positive real since it exhibits phase shift approaching -180 deg for large ω.

The positivity tests on transfer matrices may be replaced with an equivalent criterion applicable to systems in state variable form.[17]

Test for Positive Real System

Let a system with transfer matrix $G(s)$ have a minimal realization with

$$\dot{\underline{x}} = F\underline{x} + G\underline{u}, \qquad \underline{y} = H\underline{x} + D\underline{u} \tag{2}$$

Then $G(s)$ is positive real if and only if there exists a symmetric positive definitive matrix P and matrices W_0 and L such that

$$PF + F^TP = -LL^T \tag{3a}$$

$$PG = H^T - LW_0 \tag{3b}$$

$$W_0^T W_0 = D + D^T \tag{3c}$$

For a given realization, Eq. (3c) determines W_0 to within an arbitrary orthogonal transformation [if W_0 satisfies Eq. (3c) and Q is an orthogonal matrix, then (QW_0) also satisfies Eq. (3c)]. In the normal case where $D=0$, then $W_0=0$. The determination of L and P to satisfy Eqs. (3a) and (3b) is a nontrivial task; hence to test a system for positive realness may be quite difficult using this approach. Recently, Wang et al.[19] introduced an algorithm that provides such a construction.

An alternate problem that we encounter in control design is to construct a positive real system given certain fixed elements. For example, we may be given a matrix F and either G or H and seek the remaining matrix required to impose positive realness (assume $D=0$). Such a problem is solved fairly easily by choice of an L such that (F^T, L) is controllable. This guarantees a positive definite P (F must, of course, be stable) that can be easily obtained from Eq. (3a). Given G or H, the remaining matrix can be obtained from Eq. (3b).

Stability of Positive Real Feedback Systems

The importance of positivity is due to the strong stability theorems for feedback systems with positive transfer functions. This theorem was stated by Popov[20] as the following:

Theorem 1: Given the square transfer matrices $G(s)$ and $H(s)$ of equal dimension, the feedback combination of these blocks is asymptotically stable if one of the transfer matrices is strictly positive real and the other is at least positive real.

Note in Popov's notation that positive realness is equivalent to hyperstability.

Application to Structural Control

A flexible structure may be modeled as having a large (theoretically infinite) number of vibration modes that govern the motion of the structure. In practice, only a few modes are modeled accurately; the remaining modes have uncertain frequency, damping, and mode shape. The important fact to observe about the structure-control problem is the following: *the transfer function matrix between collocated velocity sensor force actuator pairs is positive real.* A proof of this can be found in Benhabib et al.[5] and will not be repeated here. This implies that for ideal velocity sensors and force actuators, any strictly positive real feedback function can stabilize the structure. This is independent of any knowledge or model of the flexible modes. In a realistic application, we demand more than just stability, and for this an appropriate model is required. The important stability result, however, is that for a positive real control design the model uncertainty cannot cause an instability with loop closure. The primary difficulty in applying this concept is the strict phase requirement of the positive real assumption. In reality no physical system can meet these requirements exactly, and to consider true stability we consider the effect of deviations from the positive real assumption.

III. Robustness Results

In this section we review the robustness results of Doyle and Stein,[8] Doyle,[9] and Safanov et al.[10] for unstructured perturbations and show how these results can be integrated with the positivity concepts to yield an improved stability result. The problem considered by Doyle is based on the Nyquist stability criteria that for single-input/single-output (SISO) systems can be phrased in terms of encirclements of the origin by the return difference function $(1+GH)$ evaluated along the $j\omega$ axis (i.e., the Nyquist contour). To ascertain the stability in the presence of unmodeled changes in the system transfer matrix, we consider a perturbation to the conventional feedback system where the actual forward transfer function $G_{act}(s)$ differs from the model $G(s)$ by the multiplicative relation

$$G_{act}(s) = \left[I + L(s)\right]G(s) \tag{4}$$

The multiplication perturbation $L(s)$ represents changes in magnitude and phase of the actual system from the nominal design model. A sufficient condition for the perturbed system to be stable is

$$\det\left[I + (I+\epsilon_0 L)GH\right] \neq 0 \qquad \text{for all} \quad 0 \le \epsilon_0 \le 1 \tag{5}$$

To relate this easily to the size of L requires recourse to the singular values of the return difference matrix. Now for Eq. (5) to hold we can equivalently require that

$$\underline{\sigma}\left[I + (I+\epsilon_0 L)GH\right] > 0 \tag{6}$$

for all ϵ_0 where $\underline{\sigma}(\ )$ indicates the minimum singular value. Using various singular value relations, Doyle showed that Eq. (6) implies the sufficiency condition

$$\underline{\sigma}\left[(I+GH)(GH)^{-1}\right] = \underline{\sigma}(I+GH)^{-1} > \text{Lm}(\omega) \tag{7}$$

or

$$\bar{\sigma}\left[GH(I+GH)^{-1}\right] < \frac{1}{\text{Lm}(\omega)} \tag{8}$$

where

$$\text{Lm}(\omega) > \bar{\sigma}(L) \tag{9}$$

is an upper bound on the magnitude of the perturbation $L(s)$. Equation (8) represents a fundamental bound on the size of perturbation allowed for the multi-input/multi-output (MIMO) system to maintain stability. This statement is a precise quantification of a familiar SISO result, namely, that the closed-loop gain should be small if the perturbation in $G(s)$ is

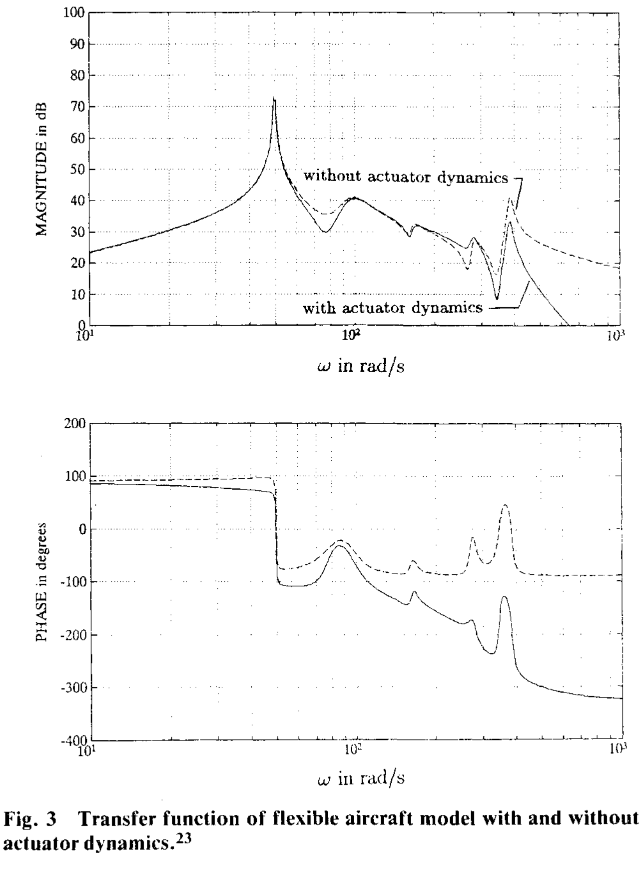

Fig. 3 Transfer function of flexible aircraft model with and without actuator dynamics.[23]

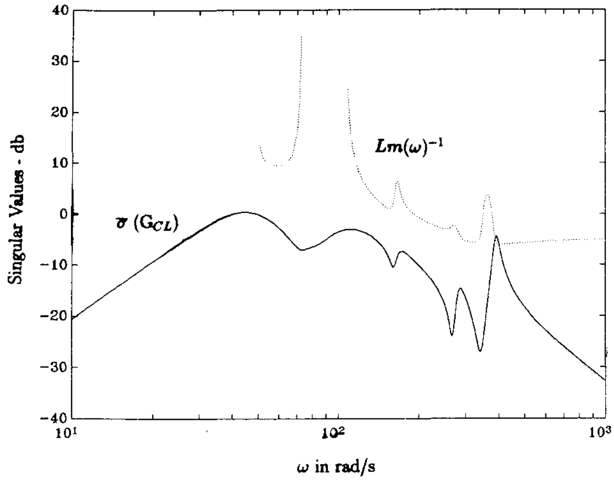

Fig. 4 Nominal closed-loop transfer function for flutter problem and actuator perturbation.

likely to be large. It is precisely at this point that the application of this robustness result to the structural control problem breaks down.

IV. Combined Positivity-Robustness Theorem

When applied to a typical modal perturbation problem, the maximum singular value of the perturbation matrix $\bar{\sigma}(L)$ will have numerous peaks corresponding to these shifted modes. Because of the relatively low frequency of these perturbations, application of the singular value bound of Eq. (8) to a modal uncertainty problem is useless. For example, a standard test model for flexible system control design is the tetrahedral truss model referred to as Draper I (see Ref. 21). A model of the Draper I structure is shown in Fig. 1. The system possesses 12 degrees of freedom with 6 inputs and 6 sensors. The nominal frequencies range from about 1 to 10 rad/s. Randomly perturbing the truss elements (areas and mass) by $\leq 5\%$ produces a different set of frequencies and mode shapes. The singular value plot $\mathrm{Lm}(\omega)$ for one typical perturbation is shown in Fig. 2. Each double peak in this plot corresponds to a slightly displaced modal frequency. The classic singular value robustness requirement is that the closed-loop transfer function must be below the inverse of this curve. This is clearly inappropriate, considering the height of these peaks. (Note that we have arbitrarily assumed a nominal modal damping of 0.2% and have not perturbed this number.) In spite of the inability to satisfy the singular value requirement, a positive real control design with ideal actuators is guaranteed to be stable since even for the perturbed model the positive real stability theorems are satisfied.

For positive real control, the primary stability question comes when additional error sources are considered, in particular those modeling errors that invalidate the positive real assumption. Most commonly these may be attributed to actuator dynamics. Other sources of error could be included as, for example, sampling delays in a digitally controlled system. The key to application of the Doyle robustness result is to evaluate the singular value bounds on the deviation from positivity, not on the deviation from an a priori model. By using this philosophy, extended stability results can be obtained for a class of realistic problems.

The combined application of the positivity-robustness theorem can be stated as follows:

Theorem: Define the perturbation matrix $L(s)$ as the multiplicative deviation from the positive real condition for the feedback system with forward function $G(s)$ and feedback function $H(s)$. The closed-loop system is stable if the singular value inequality in Eq. (8) holds.

The proof of this theorem follows almost trivially from a careful, sequential application of the positivity theorem, and then the Doyle robustness theorem. Consider the design model $G(s)$ to be positive real, and the actual $G_{act}(s)$ to be a perturbation from $G(s)$ given by

$$G_{act}(s) = \left[I + L_{tot}(s)\right] G(s)$$

where the total perturbation matrix is given by

$$I + L_{tot}(s) = \left[I + L(s)\right]\left[I + L_{pr}(s)\right]$$

and where $L_{pr}(s)$ is a perturbation such that $[I + L_{pr}(s)]G(s)$ is positive real, and $[I + L(s)]$ is the deviation from positivity. Now if $L(s) = 0$, then the system is guaranteed to be closed loop stable for any strictly positive real feedback function $H(s)$. The Doyle result [Eq. (8)] can then be applied to this stable system to verify stability with nonzero $L(s)$.

Caution must be used in applying this theorem for arbitrary modal perturbations since the inequality in Eq. (8) can only be evaluated using a modeled $G(s)$, not necessarily the actual $G(s)$. In particular, if high frequency modes have arbitrary small damping, this extension will be invalid. This result is significant, however, in that it gives verification of the extended range of validity of positive real control design.

This result can be directly applied to continuous systems with actuator dynamics by modeling the actuator perturbation as the $L(s)$ in the robustness theorem. We have also applied this result to digital systems by approximating the effect of digitization on the continuous frequency response as a pure phase delay. This approximation is not sharp, however, and cannot be used to establish conservative boundaries for the stability problem. Nevertheless, this result can be an important technique to establish stability for problems with various perturbation models.

V. Application

Using a controller designed with the positive real method, the actuator effects and the computational delays can be modeled as a high frequency perturbation $L(j\omega)$. Modal perturbations due to uncertainties caused by mass and stiffness variations are not included in this perturbative model. Applying the fundamental robustness result in Eq. (8) to the nominal closed-loop design implies in a physical sense that the stability of the controlled structure can be guaranteed if the controller bandwidth is sufficiently low, and if there is enough high frequency rolloff that the phase and magnitude perturbations occur sufficiently outside the nominal system bandwidth. Formally we require also that outside this bandwidth the inherent damping of any unmodeled modes must be large enough that the prod-

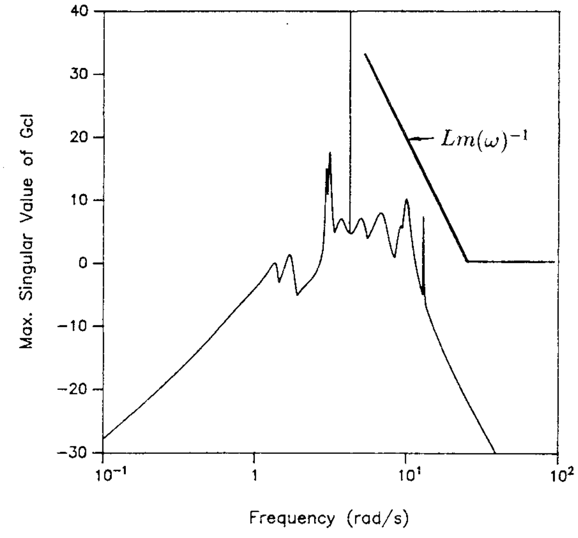

Fig. 5 Singular values for Draper I nominal closed-loop design model and Lm(ω) for actuator model.

uct of $GH(j\omega)$ is still small. In our simulation examples, this does not seem to present any difficulties, and, in fact, with damping ratios characteristic of these high frequency modes (≈ 0.02–0.10), our results have been applied with a high degree of success.

We demonstrate this methodology with several examples, including an aircraft flutter control problem, control of the Draper I model truss structure with actuator dynamics, and finally an experimentally controlled beam demonstrated in the laboratory.

Example 1: Aircraft Flutter Problem

A simple example demonstrating this concept was shown by Slater et al.[22] using an aircraft flutter problem originally formulated by Takahashi and Slater.[3] In this example the nominal transfer function between sensor and control was almost positive real if actuator dynamics were neglected. The presence of the actuator added significant phase shift, destroying the positivity at the high frequency range (see Fig. 3). Nevertheless, a positive real design that neglected the actuator was successful in controlling the structural oscillations.

This success was justified initially using heuristic arguments based on bandwidth. Using the positivity-robustness theorem allows us to guarantee the stability of this control design for both the nominal model and a perturbed model. In this and in many other cases, a reasonable first assumption is to model the deviation from positivity for the vehicle transfer function strictly as a phase change, i.e.,

$$I + L = e^{j\phi} I \tag{10}$$

hence

$$\mathrm{Lm}(\omega) = 2\left|\sin\left(\frac{\phi}{2}\right)\right| \tag{11}$$

which clearly is bounded by

$$\begin{aligned}\mathrm{Lm}(\omega) &< 2\\ &< 6\ \mathrm{dB}\end{aligned} \tag{12}$$

Note that the inequality in Eq. (12) is only applied for frequencies outside the positive real range and that the more precise bound in Eq. (11) may be applied if desired. Combining the

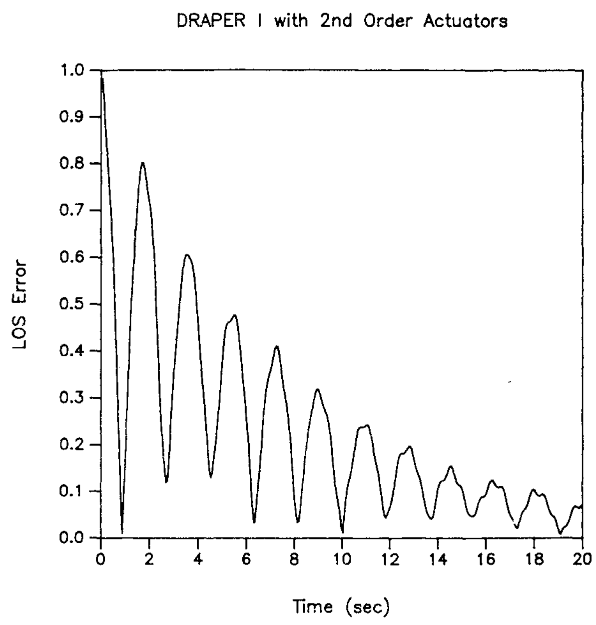

Fig. 6 Time response of Draper I model including a second-order actuator showing stable response.

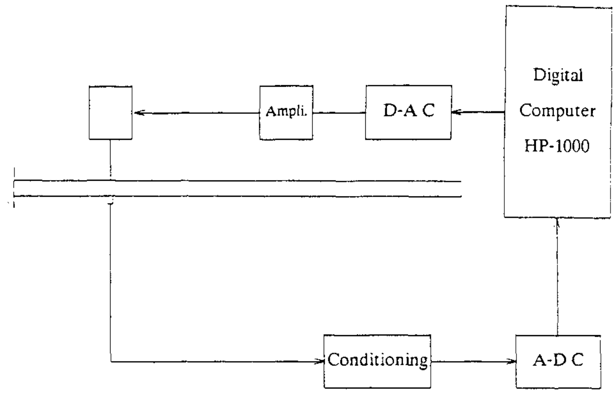

Fig. 7 Setup of laboratory beam control experiment.

system and controller models, the nominal closed-loop plant for the flutter example is shown in Fig. 4. Applying the perturbation in Eq. (11) to the closed-loop plant as shown by the dashed line in Fig. 4, we see that the criterion in Eq. (8) is satisfied (except for a small intersection at a resonant peak), giving a sufficiency condition that *guarantees* stability for this problem. This stability had previously been verified by a numerical procedure for the given modal data, but in fact we can now guarantee stability for additional modal variations from the base positive real model.

Example 2: Analytical Study of the Draper I Structure

For the Draper I structural model previously mentioned, the normal singular value inequality bound in Eq. (8) clearly cannot be satisfied for the type of modal uncertainty depicted in Fig. 2. Using a positive real control design, we know, however, that such variations will not harm the closed-loop stability of the nominal design. We have chosen a nominal design here from Ref. 22, and the singular value plot $\bar{\sigma}[GH(I+GH)^{-1}]$ is shown in Fig. 5. We now assume that the actual system has actuation dynamics modeled by the second-order transfer function

$$G(s) = \frac{g\omega s + \omega^2}{s^2 + g\omega s + \omega^2} \tag{13}$$

Using $g = 1.4$ and a natural frequency of 30 rad/s, the resultant perturbation plot is superimposed in Fig. 5. Again this plot demonstrates satisfaction of the robustness condition in Eq. (8) everywhere (except for one zero damping peak introduced by an unobservable mode). Hence, from a theoretical point of view, this positivity design can be guaranteed to be stable even with significant modeling errors in the modal model. A representative time simulation of the actual truss structure (12 modes, all with zero damping) with a controller based on a reduced-order model (3 modes) and with modeling errors introduced by random perturbations of the truss elements is shown in Fig. 6. As predicted by the stability theorem, this system is stable, and in this case, performance is not degraded by the presence of the actuator.

Example 3: Laboratory Beam

A third example involves the laboratory control of a simple cantilever beam (see Fig. 7) using a fixed mechanical shaker as the controller device. The controller includes compensation for the shaker so as to introduce negligible phase shift in the control bandwidth. The control is implemented using accelerometer inputs. These are sampled using an HP digital computer, integrated to produce a velocity signal, and a (continuous) positive real controller design is implemented in discrete form within the computer system. The robustness results are applied using the continuous formulation. The effect of the

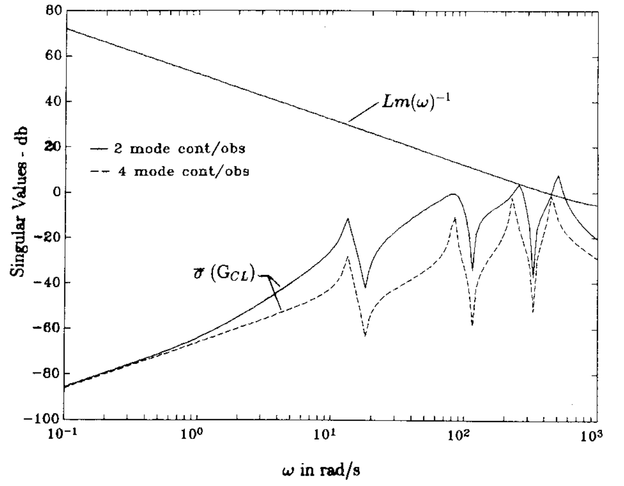

Fig. 8 Singular value plot for nominal closed-loop beam control system, Lm(ω) for nominal time delay.

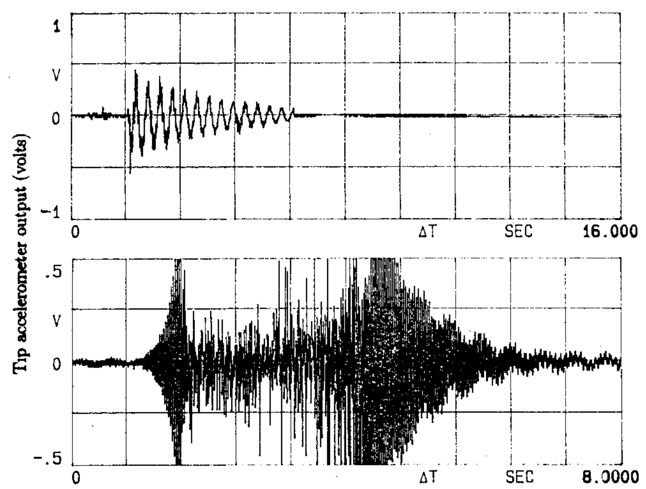

Fig. 9 Time response for beam experiment: a) two-mode controller; b) four-mode controller.

digital controller was modeled by approximating the digital system as introducing a time delay:

$$I + L(s) \approx e^{-(sT/2)} \tag{14}$$

applied to the continuous system plus controller. This approximation can be considered valid at frequencies significantly below the Nyquist frequency $\omega_s/2 = \pi/T$.

In this problem, actuation dynamics were neglected completely, and only the approximation in Eq. (14) was used to model the perturbation. Note that one advantage of the positive real theorem is that stability can normally be assured for a multivariable design even if one (or more) of the scalar feedback paths is broken due to a failed actuator or sensor. In the laboratory two controller designs were tested. The first was an SISO design based on a two-mode model of the beam. The second controller was based on a four-mode beam model and in addition was a two-input/two-output design. In the experiment, only one shaker was available, so this latter controller was used with one of the paths failed. In this way it acted similarly to the SISO design, except that four modes are considered in the design rather than two. The positive real controllers were designed using a pole placement algorithm as described in Ref. 24 such that the positivity criteria of Eq. (3) are satisfied. The singular value plots for these cases are shown in Fig. 8. Using a standard discretization and zero-order hold assumption, the digital implementation of the four-mode continuous controller design is predicted to go unstable at a sampling rate of about 5 ms (as predicted by an eigenvalue analysis). Similarly, the two-mode controller is predicted to become unstable at a sampling rate of about 10 ms. Using the approximate phase perturbation in Eq. (14) produces the unmodeled perturbations as shown by the lines in Fig. 8. The singular value plot for this case shows that both controllers are near the limiting stability boundary. Since the phase approximation is nonconservative at these high frequencies, stability is not guaranteed and suggests that additional analysis is required. In the actual experimental test, the two-mode controller was stable at a sampling rate of about 3 ms (see Fig. 9). This was the fastest sampling possible for the computer setup, and slower sample rates were not investigated for this controller. The four-mode controller design required over 5 ms per cycle and was unstable (this instability was also predicted by an eigenvalue analysis.) We are currently investigating application of these principles to a digital design to see if these robustness bounds can be applied in a more general sense for the digital problem.

V. Conclusion

This paper shows that the use of unstructured singular value tests can be combined with a positive real control design to improve stability results for systems with uncertainties in the modal parameters and with multiplicative uncertainties due to actuator and sensor dynamics. The effect of high frequency actuator and sensor dynamics can be included in a positive real design using this approach. Caution must be exercised when using the continuous theory to model the computational delays produced by a digital controller.

These results should be a valuable aid in control design for large space structures and other flexible systems with significant structure control interaction.

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